ISSN: 2710-0723

Research on Sustainable Tourism Development based on Multivariate Model Optimization

Churui Liang*
Faculty of Science and Technology, BNU-HKBU United International College, Zhuhai, China
*Corresponding author: t330036076@mail.uic.edu.cn

Abstract

In order to promote the sustainable development of tourism in Juneau, the article constructs a series of models to balance multiple factors in tourism development. By integrating the data of the past 15 years, a two-period moving average method is used to build a model of tourist numbers; the Pearson correlation coefficient is used to determine the strong positive correlation between residents' income and the number of tourists, so that the income model is abstracted as a linear function of the residents' model; and simplified differential equations are used to simulate the process of glacier degradation and build an environmental pressure model. Further, the Tourist Happiness Index (THI), Resident Happiness Index (RHI) and Sustainable Development Index (SDI) are developed to quantify the above models. Considering the constraints of environment, infrastructure, number of tourists and hidden financial expenditures, the TOPSIS model was applied to screen the relative optimal strategies. In addition, the social satisfaction index (SSI) was constructed using principal component analysis (PCA), and the tourism economic benefit function (EBT) was introduced under the Nash equilibrium framework, and the optimal sustainable development model was obtained by taking the maximization of "SSI + SDI + EBT" as the objective function, which provided the sustainable development of Juno tourism with theoretical support and decision-making basis.

Keywords

Tourism Sustainability; Predictive Model; Optimization Strategy; TOPSIS Model; Nash Equilibrium.

1. Introduction

At a time when tourism is booming globally, many regions see it as an important engine of economic growth, and Juneau is a prime example. However, Juneau faces the challenge of balancing economic growth, quality of life, and ecological protection when developing tourism. The traditional tourism development model focuses on a single goal and ignores the linkage of various factors, which makes it difficult to realize sustainable development [1].

Therefore, this paper focuses on the actual situation in Juneau and constructs a series of models to quantify the key factors, such as residents' income, the number of tourists and environmental pressure. By optimizing the model and setting multiple constraints, we search for the optimal strategy for sustainable tourism development, which is expected to provide a reference for similar regions [2].

ISSN: 2710-0723

2. Building Basic Model

2.1. Tourist Number Model

The number of tourists is a critical variable for sustainable tourism development. By integrating data from the past 15 years, we have developed a model based on a two-cycle moving average. The formula is as follows:

$$\mathrm{MA}_2(t) = \frac{X(t)+X(t-1)}{2} \tag{1}$$

2.2. Income Model

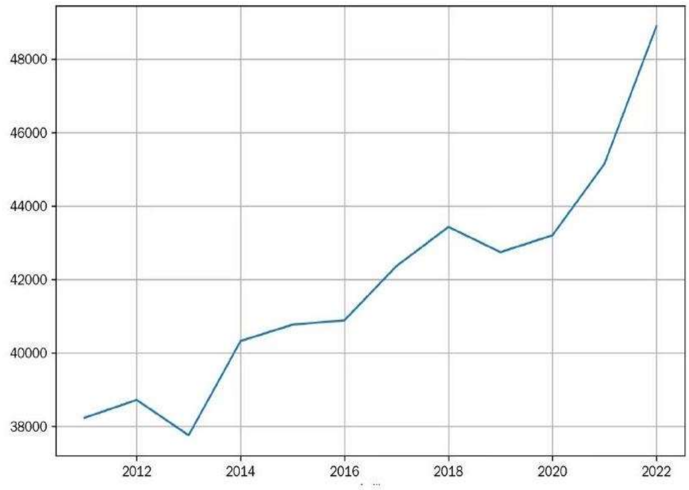


Figure 1. Income Trend

As shown in Figure 1, we find that the personal income of local residents is highly correlated with the number of tourists. Therefore, we used Pearson correlation coefficient to analyse this relationship in both models [3]. The formula is as follows:

$$r = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2\sum_{i=1}^{n}(Y_i-\bar{Y})^2}} \tag{2}$$

Through our calculations, the correlation coefficient r is 0.73, indicating a strong positive correlation between the two models. Therefore, we can abstract the income model as a linear function of the resident model. The formula is as follows:

$$Y = \beta X + \epsilon \tag{3}$$

2.3. Environmental Pressure Model

For the environmental pressure model, we focus primarily on the melting (degradation) process of glaciers. Therefore, we employ a simplified difference equation to simulate the degradation of the Mendenhall Glacier [4]. The change in its volume can be expressed by the following equation:

$$\frac{\Delta G(t)}{\Delta t} = -\alpha_1 T(t) - \alpha_2 V(t) - \alpha_3 S(t) + \alpha_4 M(t) + \beta \tag{4}$$

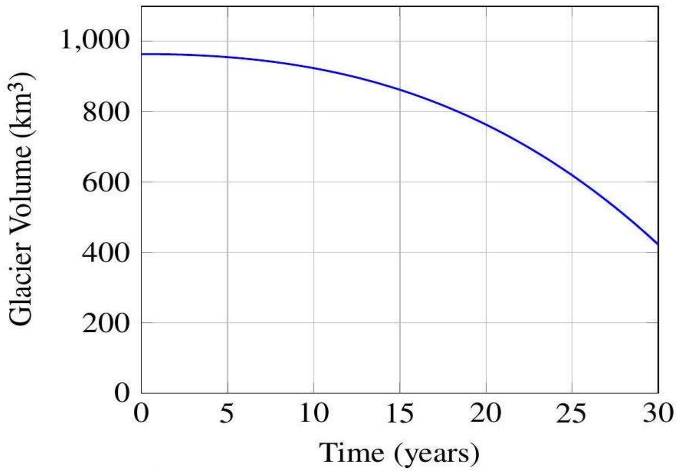


Figure 2. Glacier Volume vs. Time (Juno, Alaska)

From the figure 2, it is evident that without intervention, the glacier will degrade at an accelerating rate, posing a significant threat to Juneau's sustainable tourism.

To this end, we developed three key indicators to quantify the resident income model, the tourist volume model, and the environmental pressure model: the Tourist Happiness Index (THI), the Resident Happiness Index (RHI), and the Sustainable Development Index (SDI).

3. Optimization Factors

3.1. Residents' Happiness Index

We posit that the residents' happiness index is primarily influenced by residents' income and tourist volume. Given that Juneau is significantly impacted by tourism, we assume that residents' income largely derives from tourist expenditures. Since tourist volume is closely related to tourist consumption, we first conduct a composite analysis of the relationship between residents' income and tourist consumption.

$$\Delta V(t) \propto \frac{\beta_1}{C^2(t)} - \lambda\beta_2 e^{-\lambda C(t)} \tag{5}$$

In order to translate the variables of tourist quantity $V(t)$ and tourist consumption $C(t)$ into a single index, we can integrate them through a weighted or combined approach. Based on their interrelationships, the happiness index $H(t)$ can be modeled.

$$H(t) \propto -\gamma_1 V(t) + \text{ constant} \tag{6}$$

$$H(t) \propto \gamma_2 C(t) + \text{ constant} \tag{7}$$

Combining the above relationships, we arrive at the following comprehensive model for the residents' happiness index $H(t)$, and the trend is plotted in Figure 3:

$$H(t) = \gamma_2 \left(\frac{\beta_1}{V^2(t)} - \lambda\beta_2 e^{-\lambda V(t)}\right) - \gamma_1 V(t) + \text{ constant} \tag{8}$$

ISSN: 2710-0723

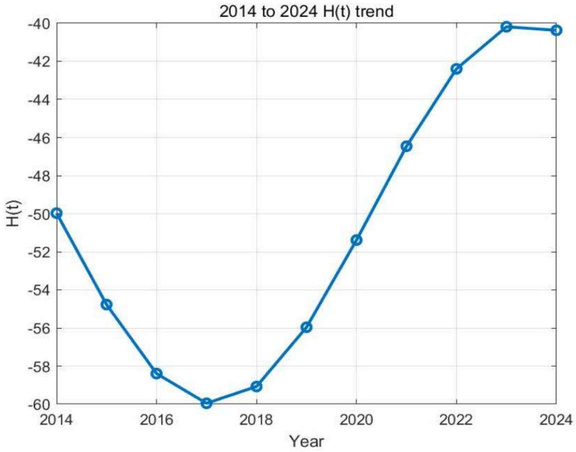


Figure 3. RHI trend

$$H_{\text{tourist}}(t) = \gamma_1 C(t) + \gamma_2 E(t) \tag{9}$$

The tourist consumption $C(t)$ can be modeled as a function of tourist quantity $V(t)$:

$$C(t) = \frac{\beta_1}{V(t)^2} - \lambda\beta_2 e^{-\lambda V(t)} \tag{10}$$

Combining the above models, the Tourist Happiness Index $H_{\text{tourist}}(t)$ can be expressed as:

$$H_{\text{tourist}}(t) = \gamma_1\left(\frac{\beta_1}{V(t)^2} - \lambda\beta_2 e^{-\lambda V(t)}\right) + \gamma_2(-\alpha_1 T(t) - \alpha_2 V(t) - \alpha_3 S(t) + \alpha_4 M(t) + \beta) \tag{11}$$

3.2. Sustainable Development Index

The Sustainability Index $SI(t)$ is determined by two main factors: environmental pressure and tourist consumption, which are described by the following models.
Combining the above models, the Sustainability Index $SDI(t)$ is given by:

$$SDI(t) = \gamma_1(-\alpha_1 T(t) - \alpha_2 V(t) - \alpha_3 S(t) + \alpha_4 M(t) + \beta) + \gamma_2\left(\frac{\beta_1}{V(t)^2} - \lambda\beta_2 e^{-\lambda V(t)}\right) \tag{12}$$

4. Establishment of Optimization Model

4.1. Environmental Protection Constraint

The environmental protection constraint is defined as the resource consumption threshold for Juneau, expressed as follows:

$$g_1(V(t), T(t), S(t), M(t)) \leq C_{\max} \tag{13}$$

Here we also constrain the melting rate of glaciers:

$$\frac{\Delta V(t)}{\Delta t} \geq -\alpha_{\max} \tag{14}$$

4.2. Infrastructure Capacity Constraints

The infrastructure capacity constraints ensure that the resources required to support the tourism activities do not exceed the maximum capacity of each infrastructure type.

The number of tourists is limited by the capacity of the transportation system:

$$V(t) \leq V_{\text{max, traffic}} \tag{15}$$

The number of tourists is limited by the available accommodation units:

$$V(t) \cdot \delta_{\text{stay}} \leq A_{\text{avail}} \tag{16}$$

4.3. Tourist Quantity Constraint

The number of tourists cannot exceed the environmental carrying capacity of the region:

$$V(t) \leq V_{\max} \tag{17}$$

$V(t)$ is the number of tourists at time t. $V_{\max}$ is the maximum tourist capacity.

4.4. Fiscal Recessive Expenditure Restraint

Regarding the hidden financial expenditures of Juneau, we focus primarily on the tourism industry, with particular emphasis on infrastructure construction and maintenance. These expenditures are assumed to have a positive correlation with both the annual number of tourists V(t) and tourist consumption C(t), expressed as follows:

$$F_{\text{infra}}(t) = \gamma_1 \cdot V(t) \cdot C(t) \tag{18}$$

The second category is environmental protection expenditure, which we posit is directly proportional to the rate of glacier melting. This relationship is expressed as follows:

$$F_{\text{env}}(t) = \gamma_2 \cdot \frac{\Delta V(t)}{\Delta t} \tag{19}$$

The total hidden fiscal expenditure in Juneau City due to tourism is:

$$F_{\text{hidden}}(t) = \gamma_1 \cdot V(t) \cdot C(t) + \gamma_2 \cdot \frac{\Delta V(t)}{\Delta t} \tag{20}$$

The total hidden fiscal expenditure must be constrained by a maximum allowable expenditure $F_{\max}$:

$$F_{\text{hidden}}(t) \leq F_{\max} \tag{21}$$

5. Topsis Model - the Exponential Optimal Solution

Under the TOPSIS model, we construct a decision matrix where each row represents an alternative and each column represents a target indicator (RHI, THI, and SDI).

ISSN: 2710-0723

We define the decision matrix X as an $m \times n$ matrix, where m represents the number of alternative solutions, and n represents the number of criteria. Each element x_{ij} represents the score of the i-th alternative on the j-th criterion [5].
The decision matrix is as follows:

$$X = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ \vdots & \vdots & \vdots \\ f_{m1} & f_{m2} & f_{m3} \end{pmatrix} \tag{22}$$

The normalized decision matrix X' is calculated as:

$$X'_{ij} = \frac{X}{\sqrt{\sum_{i=1}^{m} f_{ij}^2}} \tag{23}$$

The ideal solution (A^+)and the negative ideal solution (A^-)are defined as:

$$A^+ = (\max f_1, \max f_2, \max f_3) \tag{24}$$

$$A^- = (\min f_1, \min f_2, \min f_3) \tag{25}$$

The Euclidean distance from each alternative to the ideal solution D_i^+ and negative ideal solution D_i^- are computed as:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(f_{ij}^+ - f_{ij}\right)^2} \tag{26}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(f_{ij}^- - f_{ij}\right)^2} \tag{27}$$

The closeness coefficient C_i is then calculated as:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{28}$$

Subsequently, the four major dynamic constraint ranges mentioned above are introduced. Different schemes are screened within the decision matrix, and the objective function is adjusted according to the constraint conditions. Finally, the maximum value of C_i is calculated, and the corresponding countermeasures are identified as the optimal policies under these conditions.

According to the results of the calculations, three optimal strategies have been identified, yielding values of 0.9986, 0.9769, and 0.9511 respectively.

These strategies maximize the closeness coefficients for the three indices (RHI, THI, and SDI) in a relative sense. Under these strategies, it was observed that the sustainability index achieved its maximum value under environmental protection constraints, while the residents' happiness

ISSN: 2710-0723

index reached its peak under constraints related to tourist numbers; conversely, the tourists' happiness index remained generally at a low level.

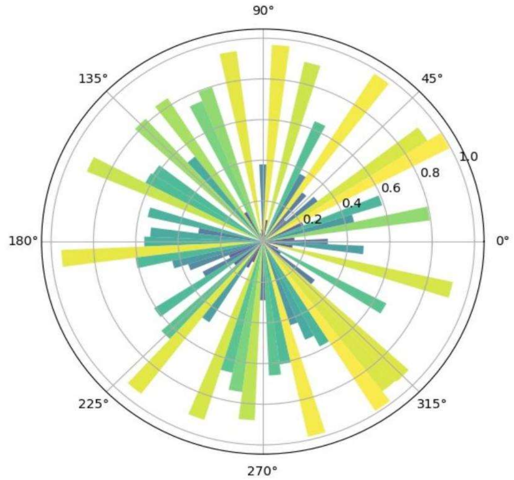


Figure 4. TOPSIS Result

In this case, we achieved a relatively optimal solution.

The composite index derived from the fitting of the three individual indicators peaks when the environmental protection limits and the number of tourists approach their respective limits. However, this level of optimization is not sufficient to support sustainable development. Therefore, in subsequent analyses, we will adopt stricter fitting constraints to maximize economic benefits while maintaining the integrity of the three key indicators, as shown in Figure 4.

6. Build Sustainable Models

6.1. PCA

We constructed a new function by fitting RHI and THI and named it the Social Satisfaction Index [6].

The covariance matrix Σ of these two variables is defined as:

$$\Sigma = \begin{pmatrix} \mathrm{Var}(THI) & \mathrm{Cov}(THI, RHI) \\ \mathrm{Cov}(RHI, THI) & \mathrm{Var}(RHI) \end{pmatrix} \tag{29}$$

The covariance between THI and RHI is calculated as:

$$\mathrm{Cov}(THI, RHI) = \frac{1}{n-1}\sum_{i=1}^{n} (THI_i - T\bar{H}I)(RHI_i - R\bar{H}I) \tag{30}$$

We begin by decomposing the covariance matrix Σ as follows:

$$\Sigma = V\Lambda V^T \tag{31}$$

The eigenvalues λ_i and eigenvectors v_i satisfy the following relationship:

$$\Sigma v_i = \lambda_i v_i \tag{32}$$

ISSN: 2710-0723

Next, we select the principal component k. The principal components are ordered based on the size of their corresponding eigenvalues. The component with the largest eigenvalue is the most significant, as it explains the largest proportion of the data's variance.

To compute the proportion of variance explained by each principal component, we use the following formula:

$$\text{Variance Proportion } = \frac{\lambda_i}{\sum_{j=1}^{p} \lambda_j} \tag{33}$$

This formula expresses the proportion of the total variance explained by the i-th principal component.

To evaluate the goodness of the fit for a given k, we use the following approximation formula:

$$\frac{\frac{1}{m}\sum_{i=1}^{m} \left\| x^{(i)} - x_{\text{approx}}^{(i)} \right\|^2}{\frac{1}{m}\sum_{i=1}^{m} \left\| x^{(i)} \right\|^2} \leq t \tag{34}$$

By evaluating the variance proportion for different values of k, we determine that $k = 1$ maximizes the variance explained, yielding the best fit.

So we can get this figure 5:

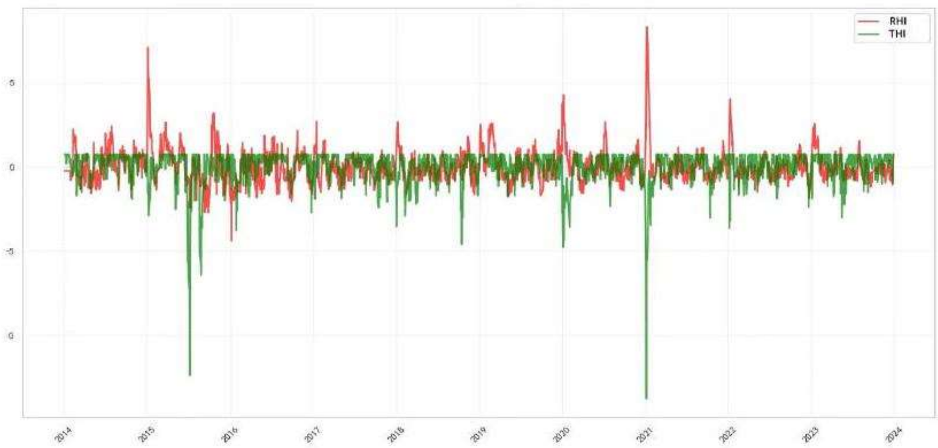


Figure 5. RHI VS THI

In this way, we defined Social Satisfaction Index is:

$$SSI = \frac{1}{e^{3(RHI - THI)}} \tag{35}$$

6.2. Nash Equilibrium

Under the Nash Equilibrium framework, we examine three key dimensions of balance: the Social Satisfaction Index (SSI), the Sustainable Development Index (SDI), and the Economic Benefits of Tourism (EBT) [7].

Here, we introduce a new function: the tourism economic benefit function.

Based on our previous assumptions, for the entire city of Juneau, the economic benefits are primarily reflected in consumer expenditures. Additionally, we incorporate ecological conservation fees (such as carbon footprint neutralization fees) and excise taxes (such as hotel taxes) imposed by the municipality on visitors. This enhances the realism of the function and improves the accuracy of its data simulation.

To model the overall economic benefit of tourism in Juneau City, we consider the following components:

Consumer Spending: Tourists' total expenditure $C_{\text{total}}^{(i)}$ consists of accommodation, dining, transportation, etc.

Ecological Protection Fees: The municipality charges tourists an ecological protection fee $E_{\text{eco}}^{(i)}$, such as carbon footprint neutralization fees.

Consumption Taxes: The municipality imposes a consumption tax $T_{\text{tax}}^{(i)}$, including hotel taxes and dining taxes.

The total economic benefit from tourism in Juno City is represented by the following function:

$$E_{\text{tourism}} = \sum_{i=1}^{N} \left(C_{\text{total}}^{(i)} - E_{\text{eco}}^{(i)} - T_{\text{tax}}^{(i)}\right) \tag{36}$$

The total expenditure of each tourist can be broken down as follows:

$$C_{\text{total}}^{(i)} = C_{\text{hotel}}^{(i)} + C_{\text{food}}^{(i)} + C_{\text{transport}}^{(i)} \tag{37}$$

The ecological protection fee is proportional to the total expenditure:

$$E_{\text{eco}}^{(i)} = \alpha \cdot C_{\text{total}}^{(i)} \tag{38}$$

where α is a constant reflecting the ecological impact per unit of expenditure.

The consumption tax is also proportional to the total expenditure:

$$T_{\text{tax}}^{(i)} = \beta \cdot C_{\text{total}}^{(i)} \tag{39}$$

Thus, the tourism economic benefit function becomes:

$$E_{\text{tourism}} = \sum_{i=1}^{N} \left[\left(C_{\text{hotel}}^{(i)} + C_{\text{food}}^{(i)} + C_{\text{transport}}^{(i)}\right) - \left(\alpha \cdot \left(C_{\text{hotel}}^{(i)} + C_{\text{food}}^{(i)} + C_{\text{transport}}^{(i)}\right)\right) - \left(\beta \cdot \left(C_{\text{hotel}}^{(i)} + C_{\text{food}}^{(i)} + C_{\text{transport}}^{(i)}\right)\right)\right] \tag{40}$$

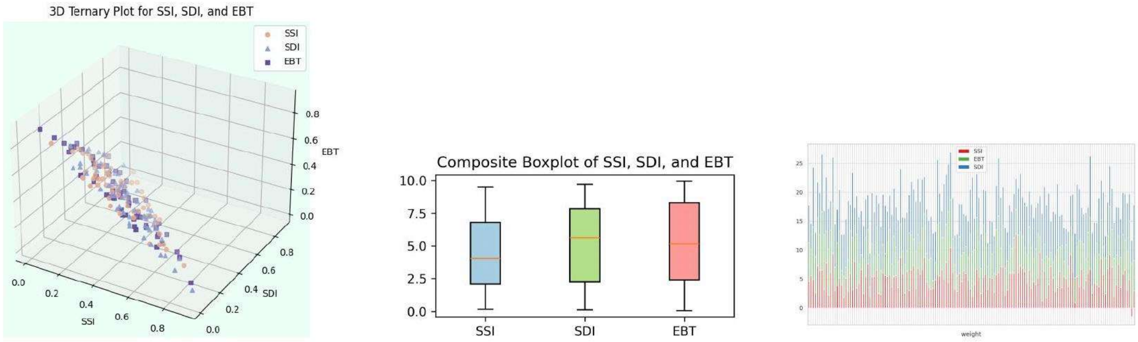
So our objective function is:

$$\max\mathcal{F} = \max(\text{SSI} + \text{SDI} + \text{EBT}) \tag{41}$$

As shown in Figure 6, it is evident that the comprehensive value of $\mathcal{F}$ is primarily influenced by the Social Satisfaction Index (SSI) and the Economic Benefits of Tourism (EBT). To maintain a high Sustainable Development Index (SDI), which is crucial for forming a sustainable development model, it is inevitable that the number of tourists will be reduced, thereby affecting the Tourism Happiness Index (THI) and overall tourism revenue. After applying the

ISSN: 2710-0723

final dynamic comprehensive constraints, we obtained a maximum value of $\mathcal{F}$ at 27.3493, leading to the optimal sustainable development model under these social conditions.



(a) Box (b) SSI vs SDI vs EBT (c) $\mathcal{F}$ Value

Figure 6. NASH

7. Conclusion

The series of models constructed in this study around the sustainable development of tourism in Juneau effectively integrate multiple factors and provide a comprehensive quantitative analysis framework for regional tourism planning. From the perspective of the basic model, the relationship between the number of tourists, residents' income and environmental pressure is accurately portrayed, revealing the intrinsic connection between the key elements of tourism, which provides a solid foundation for the subsequent development of optimization strategies.

In terms of optimization factors and model construction, the quantification of residents' happiness index, tourists' happiness index and sustainable development index, combined with the solution of TOPSIS model under multiple constraints, successfully screened out the relative optimal strategy, and to a certain extent, achieved the balance of multi-objectives. The social satisfaction index constructed by PCA and the optimal sustainable development model derived from the introduction of the tourism economic benefit function under the Nash equilibrium framework further enhance the knowledge and planning ability of tourism development from a comprehensive perspective.

However, the study also has some limitations. Some of the parameter assumptions and data processing methods in the model may deviate from the actual situation, and the impacts of external contingencies (e.g., natural disasters, global public health events, etc.) on the model have not been adequately considered. Future studies may consider incorporating more real-time data and complex influencing factors to improve the adaptability and accuracy of the model. Overall, this study provides valuable ideas and methods for the sustainable development of the tourism industry in Juneau and similar areas, and is of positive significance in promoting the scientific development of the tourism industry.

References

[1] ZHANG Xiaoling,WANG Jianing,TEDAN Drolgar. Research on the impact of coupled development of tourism and new urbanization on the income gap between urban and rural residents in Linzhi City[J]. Modern Marketing(Lower Decade),2025,(02):101-103.DOI:10.19932/j.cnki.22-1256/F. 2025.02.101.

[2] Li Li-Ling. Theory and practice of sustainable tourism development and ecological civilization planning[J]. Heilongjiang Environmental Bulletin,2025,38(01):38-40.

[3] Feng Xingjin. Exploration of the assessment method of usual grades based on Pearson's correlation coefficient[J]. Gansu Education Research,2025,(02):114-117.

ISSN: 2710-0723

[4] CHEN Minfeng,LIANG Meili,GAO Zongsheng. Existence of subpure solutions for a class of nonlinear differential-difference equations[J/OL]. Science China:Mathematics,1-16[2025-02-19].http://kns.cnki.net/kcms/detail/11.5836.O1.20250113.0933.002.html.

[5] Sai-Sai Cheng,Feng Zhu,Bi Zhou. Evaluation of drug utilization of infliximab for injection based on weighted TOPSIS method[J/OL]. Modern Drugs and Clinics,1-6[2025-02-19].http://kns.cnki.net/kcms/detail/12.1407.R.20250218.1327.006.html.

[6] He Zhaonan,Jing Min,Han Hengtong,et al. Sparse principal component analysis-random forest algorithm combined with fluorescence spectroscopy for soil surface oil species identification[J]. Analytical Laboratory,2025,44(02):216-222.DOI:10.13595/j.cnki.issn1000-0720.2023110304.

[7] Liang ZH,Wang LF,Li JL. Optimized control method for optical-storage coupled hydrogen production system based on Nash equilibrium[J]. Power System Automation,2025,49(03):125-134.