

Research on Category Management and Inventory Control Strategies of Multi Regional Network Retail Enterprises under Alternative Demand

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Abstract

This article proposes a data mining based substitution rate estimation method to address the issue of product substitution demand in online retail. By introducing the Apriori algorithm to mine association rules between product attribute combinations and sales status, effective attribute combinations that meet support, confidence, and improvement thresholds are extracted, solving the problem of uniform nominal attribute dimensions and sequencing. On this basis, a substitution rate estimation model considering the effect of neighboring substitution and the strength of attribute combination association was constructed, avoiding the red blue bus problem in traditional models. This method provides a data-driven solution for category management and inventory control decision-making in online retail enterprises, and has important theoretical value and practical significance.

Keywords

Online Retail; Substitutability; Category Management; Inventory Control; Attribute Analysis.

1. Introduction

With the development of e-commerce, online platforms have become an important channel for residents' consumption and merchants' sales in China. In order to attract different customer groups, the category structure of online stores is more complete compared to offline stores. The increase in category depth not only generates a large amount of page maintenance and traffic costs, but also leads to increased substitutability between products. How to estimate the impact of changes in category decision-making schemes on the overall profit level and choose varieties is also one of the difficulties in the operational decision-making of online retail enterprises. In the online retail industry, the substitution effect between products is more pronounced than offline. In order to meet the needs of customers in various regions, enterprises often try to configure a rich product portfolio as much as possible to ensure that customers can ultimately make purchases in their own stores. Due to the increasing similarity between products, the substitution effect between products has become a phenomenon that must be considered, thus requiring appropriate substitution rate estimation methods.

Early research on alternative demand mainly focused on numerical simulation and comparative analysis. Smith and Aggraval (2003) [1] compared three alternative methods through simulation and pointed out that product proximity substitution is closer to the real situation compared to random substitution and substitution of a given variety; Anupindi (1999) [2] studied the impact of complete and partial substitution of products on inventory given the number of categories and the distribution of corresponding consumer groups. It was found that substitution between products reduces inventory holding costs because substitution demand is also a component of inventory consumption.

Random substitution does not conform to the real demand laws (Smith, Aggraval, 2003) [3], while subset substitution itself violates the definition of "category". Since there is no possibility of substitution between subsets, they can be treated as two independent categories directly. Therefore, there are more studies in this field that use proximity substitution and proportional substitution. However, current models in category planning have shortcomings in characterizing substitution rate calculations [4-7]. Although the MNL model, LCM model, and EDM model can all characterize the decision-making behavior of some consumers while considering the substitutability of products, they each have some shortcomings [8]. In addition, for most products, such as footwear and fashion items such as clothing, even frontline salespeople find it difficult to assign weights to the sub attributes of the products [9]. Moreover, none of the above models consider the impact of product attribute combinations on consumer behavior [10], and a feasible substitution rate estimation method needs to be proposed to compensate for the shortcomings of the above research.

The fusion decision-making model proposed in this article extracts effective product attribute combinations through association rule mining techniques, establishes a multidimensional substitution rate estimation framework, and focuses on considering the proximity substitution effect between products and the strength of attribute combination associations. In the process of model construction, we strictly adhere to the principle of data-driven approach, retaining only the statistically significant product attribute association rules to ensure the objectivity and reliability of the substitution rate parameters. By quantitatively analyzing the impact of different attribute combinations on sales status, the transformation from nominal attributes to computable indicators can be achieved, providing standardized input for subsequent decision-making. This model abandons the subjective empirical parameters in traditional category planning and constructs an alternative relationship network entirely based on actual sales data and attribute correlation features of the product.

2. Model Building

In order to avoid the red blue bus problem (where the presence or absence of selected attributes does not have a positive impact on consumer decision-making), when calculating product attribute values, it is necessary to identify all attribute sets that can significantly affect consumer decision-making based on actual sales conditions. This article combines this type of attribute into an effective attribute combination, which satisfies the association rule $X \rightarrow Y$ (where Y represents the event of "product sales status being a bestseller"). If the existence of a combination of attributes does not have a positive impact on consumer decision-making, then this combination is not an effective attribute combination. According to the definition of association rules (Agrawal, 1995) [11], effective attribute combinations that can form association rules with sales status must simultaneously meet the following three threshold conditions:

Condition 1: For a combination of attributes that can have a significant impact on sales, the ratio of the number of products that include this attribute and have recently had a positive impact on sales status to the total number of products must be greater than a certain level (minimum support level), otherwise it cannot be said that the attribute combination has universal influence.

Condition 2: For a combination of attributes that can have a significant impact on sales, the probability of the product's recent sales status being positively affected by the attribute must be greater than a certain level (minimum confidence level), otherwise it cannot be proven that the attribute combination has a significant impact.

Condition 3: The probability of positive impact on the sales status of products containing this attribute combination must be significantly improved (degree of improvement) compared to

products without this attribute combination. Otherwise, it indicates that the existence of this attribute combination has no positive impact on the sales status of the product (i.e. consumer decision-making).

In order to better describe the problem from a theoretical perspective, this article first defines it as follows:

Definition 1:

For a given product set $\{S\}$, define its attribute set $\{A\}$ as $\{attribute_1, attribute_2, attribute_3, \dots, attribute_{n-1}, attribute_n\}$. This set contains all the attributes related to products belonging to $\{S\}$, equivalent to the union of these attributes.

For a given product i , it can correspond to a sequence of attributes, namely $\{attribute_{1i}, attribute_{2i}, attribute_{3i}, \dots, attribute_{n-1i}, attribute_{ni}\}$.

The element $attribute_{ji}$ ($j \in [1, n]$, and is an integer) in the attribute series represents the value of product i in the attribute dimension j . If the value is 1, it indicates that product i contains the attribute. If the value is 0, it indicates that product i does not contain the attribute, that is, all attributes are measured by whether they are included. Thus, the attribute vectors of each product in the product set $\{S\}$ are obtained.

Based on the actual sales situation of products during the recent non promotional period of the enterprise, sales personnel will provide a status label indicating whether each product i ($i \in \{S\}$) participating in sales is a bestseller. This label will be used as the $(n+1)$ th dimensional attribute of each product, denoted as $attribute_{n+1i}$. Therefore, if $attribute_{n+1i} = 1$, it indicates that the product is a bestseller, and if $attribute_{n+1i} = 0$, it indicates that the product is not a bestseller. In fact, for salespeople, the difficulty of determining whether a non promotional product is selling well in the near future is much lower than judging the attribute itself. At this point, the attribute set $\{A\}$ becomes $\{attribute_1, attribute_2, attribute_3, \dots, attribute_n, attribute_{n+1}\}$.

Definition 2:

The attribute rule set $\{C\}$ represents all possible combinations of attributes, each of which can be represented by a set of attributes. In this article, these vectors are referred to as itemsets, and for each itemset, there is a unique sequence number TID corresponding to it. For example, in a certain itemset that only contains attribute 1 and attribute 2, if the itemset sequence number TID is 1, then there are:

$$c_{TID=1} = \{attribute_1, attribute_2\} \tag{1}$$

For an itemset with k attribute elements, it is called a k -itemset, for example, c_1 here is a 2-itemset.

Definition 3:

The proportion of best-selling products containing $c_{TID=m}$ in the product set $\{S\}$ is defined as $Support(c_{TID=m})$.

$$support(c_{TID=m}) = \frac{NUM(c_{TID=m} \cup attribute_{n+1=1})}{NUM(\{S\})} \tag{2}$$

Definition 4:

The probability that the value of $attribute_{n+1}$ in a product containing itemset $c_{TID=m}$ is 1 is $Confidence(Confidence(c_{TID=m}))$.

$$\text{Confidence}(c_{TID=m}) = \frac{\text{NUM}(c_{TID=m} \cup \text{attribute}_{n+1}=1)}{\text{NUM}(c_{TID=m})} \quad (3)$$

Definition 5:

Define the probability of the product attribute $\text{attribute}_{n+1} = 1$ in the product set $\{S\}$, that is, the probability of the product selling well is $\text{Support}(\text{attribute}_{n+1} = 1)$.

$$\text{Support}(\text{attribute}_{n+1} = 1) = \frac{\text{NUM}(\text{attribute}_{n+1}=1)}{\text{NUM}(\{S\})} \quad (4)$$

Definition 6:

Define the lift of itemset $c_{TID=m}$ as $\text{Lift}(c_{TID=m})$:

$$\text{Lift}(c_{TID=m}) = \frac{\text{Confidence}(c_{TID=m})}{\text{Support}(\text{attribute}_{n+1}=1)} \quad (5)$$

If $\text{Lift}(c_{TID=m})$ is greater than 1, it indicates that $c_{TID=m}$ has an increasing effect on the product's sales probability. Conversely, the presence or absence of $c_{TID=m}$ does not affect the product's sales. Let $\alpha_{c_{TID=m}} = \text{Lift}(c_{TID=m}) - 1$ represent the additional improvement of the attribute combination, that is, the positive impact on the product's sales status.

Therefore, the three conditions proposed at the beginning of this section can be described using three thresholds:

The specific criteria for determining the minimum support (defined as min_{sup}), minimum confidence (defined as min_{conf}), and minimum improvement (defined as min_{lift}) are as follows:

(1) If $\text{Support}(c_{TID=m}) \geq \text{min}_{sup}$, then $c_{TID=m}$ is called a frequent itemset, otherwise it is called a non frequent itemset. If $c_{TID=m}$ is a K-itemset and a frequent itemset, it is called a frequent K-itemset.

(2) For the frequent K-itemset $c_{TID=m}$, if its confidence level $\text{Confidence}(c_{TID=m}) \geq \text{min}_{conf}$, then $c_{TID=m}$ is called a strong association rule.

(3) If $c_{TID=m}$ is a strong association rule and $\text{Lift}(c_{TID=m}) \geq \text{min}_{lift}$, then $c_{TID=m}$ is considered a valid rule. At this point, the positive impact of the attribute combination on sales, $\alpha_{c_{TID=m}}$, is equal to $[\text{Lift}(c_{TID=m}) - 1]$. For each valid rule that meets the requirements, there will be a corresponding positive impact parameter value, $\alpha_{c_{TID=m}}$.

Through this method, the positive impact of the effective attribute combination $c_{TID=m}$ on product purchase can be measured. Based on the correspondence between each product attribute in $\{S\}$ and the effective attribute combination, the comprehensive attribute value of each product attribute combination can be calculated. After weighting and summing with the attribute values of other non nominal attributes obtained by traditional AHP or expert scoring methods, the overall attribute value of the product can be determined, and then the similarity and substitution rate between products can be estimated by referring to the existing literature.

At this point, the problem transforms into solving for all valid rules and their corresponding degrees of improvement. If the attribute dimension of the product is low, the above calculation is easy to complete. However, in reality, the dimensionality of product attributes is usually very high, especially for products with nominal attributes (such as footwear and clothing products), resulting in an exponential increase in the number of attribute combinations. Assuming an 18 dimensional nominal attribute set, the number of frequent itemsets that may be generated

reaches 2 to the power of 18, with approximately 260000 candidate sets. In order to effectively improve computational efficiency, this paper will use the Apriori algorithm, a commonly used association rule mining algorithm in the field of data mining, to mine effective attribute combinations.

3. Mining Methods for Effective Attribute Combinations

3.1. Attribute Combination Mining Method based on Apriori Algorithm

The Apriori algorithm was proposed by Agrawal et al. based on the idea of association rules, and it is widely used in the retail field. Retailers can analyze POS data to identify which product combinations frequently appear, and then bundle them for sale to obtain additional profits. Therefore, this type of problem regarding the association rules between itemsets is collectively referred to as the "shopping basket problem".

The so-called association rules are implicit expressions in the form of $X \rightarrow Y$, where X is called the antecedent (also known as the antecedent) of the association rule, and Y is called the consequent (also known as the successor). If the relationship between $X \rightarrow Y$ holds, it indicates that the appearance of X is related to the appearance of Y . Consistent with the assumption in the previous section, in the mining of association rules, the main reference indicators are support and confidence, while determining whether a rule is effective also depends on its degree of improvement.

This article borrows the idea of shopping basket analysis and regards the attribute information contained in SKUs as a shopping basket. The shopping basket contains many nominal attributes (among which the $(n+1)$ th dimensional attribute is the recent sales status of the product). Now we know that some SKUs' shopping baskets contain the "best-selling" label, so we hope to find all the attribute label combinations that often appear together with this label, that is, the attribute combination X that satisfies the $X \rightarrow Y$ (Y is the event of "product sales status is best-selling") rule.

The Apriori algorithm adopts a pruning method for high-dimensional itemsets, which can greatly reduce the workload of the algorithm. Here is a simple example to illustrate that the Apriori algorithm uses a two-step pruning method of "connection step pruning step" when searching for frequent itemsets:

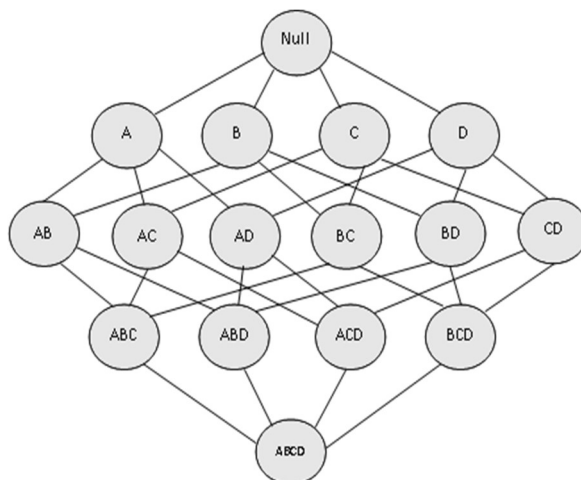


Figure 1. Potential frequent itemset network

Assuming there are four attributes in $\{A\}$ now, namely A,B,C,D, So the composition of its possible frequent itemsets is shown in the above figure. If the enumeration method is used for

computation, a total of 3 indicators from 15 itemsets need to be calculated. Namely, support, confidence, and improvement. In order to improve its algorithm performance, the Apriori algorithm first separates the calculation of support and confidence, and separates the frequent itemset mining stage.

In order to further reduce computational complexity, the Apriori algorithm fully utilizes the following two prior knowledge:

Theorem 1: A subset of frequent itemsets must be a frequent itemset.

Theorem 2: The superset of a non frequent itemset must be non frequent.

Among them, a superset refers to the set generated by adding other factors to the original combination, which can be understood as the true subset of its superset. For example, {ABCD} is a superset of {AB}, and for a superset that only has one more term than the original combination, it becomes a direct superset. For example, {ABC} is a direct superset of {AB}.

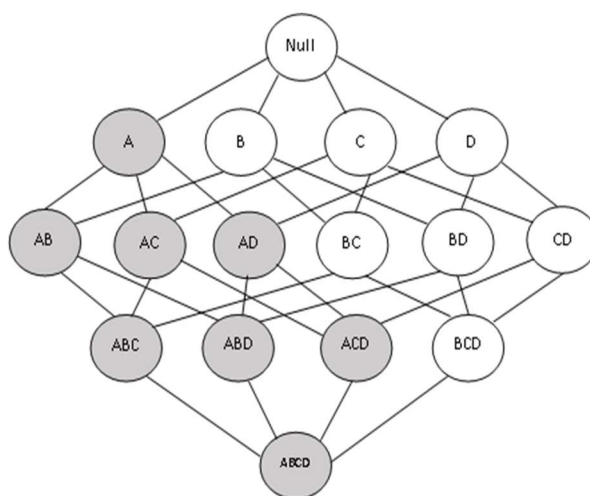


Figure 2. Pruning the superset of non frequent itemsets

Here, assuming a minimum support of 3%, the Apriori algorithm first calculates the support of 1-itemsets and then determines which 1-itemsets meet the minimum support requirement. Assuming that the calculated support of A is less than 3% and BCD meets the requirements, then A is no longer a valid rule, and the non valid rules are filled in gray in the figure. Obviously, since AB, AC, and AD are all direct supersets of A, their support must be less than or equal to A. Therefore, if A does not meet the minimum support requirement, they will also not meet the requirement, and there is no need to consider these combinations in the next stage. Eight combinations that do not meet the requirements were immediately removed from the graph, greatly reducing the workload of the algorithm.

When the Apriori algorithm is unable to find new frequent itemsets, the first step ends. Then, the support and elevation of the frequent itemsets that meet the minimum support condition are calculated to output association rules that meet the conditions.

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Generally speaking, its main algorithm process is as follows:

Step 1: Scan the dataset, calculate the support for each data item, and obtain the frequent 1-item set

Step 2: Generate candidate K-itemsets of length K through self connection from frequent K-1 itemsets

Step 3: In order to count the support of the candidate options, the dataset is scanned again to calculate the support of each candidate option in the candidate $K-1$ item set C_K , and the candidate items with at least one non frequent subset are pruned

Step 4: When no new frequent itemsets are generated, i.e. $C_K = \{\}$, the algorithm ends

Step 5: Calculate the confidence and improvement of frequent itemsets

Step 6: Output rules that meet the requirements

Thus, the problem raised in the previous section can be solved, and all effective attribute combinations that can stimulate consumers to make consumption decisions and their corresponding positive influence parameters can be obtained. In the next section, this article will use a practical case to carry out the above process.

3.2. Case Analysis

The data for this case is sourced from 772 Q2 running shoe products sold in the official Tmall flagship store of Company A in 2015 (not broken down by size). The basic information includes the attributes corresponding to these products and the best-selling labels corresponding to the SKUs during the current non promotional period.

The best-selling products, which account for 20% of the total sales, contribute more than 85% of the sales volume. Therefore, in the online retail industry, the Pareto Principle is more pronounced than in offline retail. Therefore, if we only analyze from the perspective of the order line, it is difficult to notice the key attribute differences between best-selling and non best-selling products because the sales records of non best-selling products are too few.

Due to the fact that the Apriori algorithm requires the input of minimum support and minimum confidence thresholds, the setting of this parameter directly affects the final result. Generally speaking, support and confidence cannot be achieved at the same time. The higher the support, the lower the average confidence of the rules. If both high confidence levels are set, it is likely that no effective rules will be generated. Similarly, the higher the confidence level, the lower the support of the rules. At this point, although the rules mined are strongly associated rules, they do not have universality. If a high support level is set at the same time, effective rules will not be mined.

Excessive confidence and low support can lead to the consequence of overfitting, which refers to the fact that although the rule is very correct in historical data, its accuracy may be relatively poor in practical applications. However, low confidence and high support can lead to insufficient learning, resulting in rules that are common but have poor accuracy.

In this example, the support level is set to 5% in this article. The confidence level is set to 40%, and each rule obtained under this condition can involve 5% or about 40 SKUs, which has a certain universality. The confidence level is set to 40%, which is exactly twice the proportion of best-selling products, and can ensure that the improvement of this rule is greater than or equal to 2, that is, the impact on sales is significant.

Table 1. Comparison of Rule Mining Results under Different Parameter Scenarios

Parameter scheme	minimum support	minimum confidence	average number of instances	average support	average confidence
1	1%	70%	9.14	1.19%	75.85%
2	3%	50%	26.56	3.45%	52.31%
3	5%	40%	43.68	5.67%	42.79%
4	10%	30%	93.18	12.08%	31.61%
5	20%	20%	217.53	28.21%	22.81%

Table 2. Comparison of Rule Mining Results under Different Parameter Scenarios (Continued)

Parameter scheme	minimum support	minimum confidence	average number of instances	average support	average confidence
1	1%	70%	3.75	6.38	435
2	3%	50%	2.59	4.76	41
3	5%	40%	2.11	3.97	38
4	10%	30%	1.56	3.25	40
5	20%	20%	1.13	2.32	148

In order to compare the impact of parameter settings, this article used five sets of parameter settings to compare their differences, namely (support 20, confidence 20), (support 10, confidence 30), (support 5, confidence 40), (support 3, confidence 50), and (support 1, confidence 70). The results are shown in Table 2 (operating software is Clementine 12.0).

From the mining results, it can be found that although the rules obtained from Scheme 1 and Scheme 2 have a high degree of improvement, they involve too few cases and do not have universal significance, which raises suspicion of overfitting; Although Scheme 4 and Scheme 5 involve an average of 93 and 217 SKUs respectively, their average improvement is relatively low, resulting in insufficient influence of attribute combinations on consumer decision-making. In addition, it is also possible to compare the number of rules with the average number of attributes. The more rules there are, the easier it is to locate fewer SKUs. If we set the minimum support to be smaller, it is possible to mine attribute combinations that are exactly the attributes of each best-selling product. In this case, the results obtained are invalid because the original intention of association rule mining is to mine commonalities, and attribute combinations that do not have universality are unnecessary. At this point, this article retains the attribute combinations obtained from scheme three as elements of the valid attribute set. There are a total of 38 valid rules in Plan 3, including 7 rules in 3 itemsets, 25 rules in 4 itemsets, and 6 rules in 5 itemsets. Here are the top ten attribute combinations for improvement ranking:

Table 3. top ten attribute combinations for promotion

Rule serial number	Preceding item (attribute combination)	Instance	support%	confidence%	promotion	rule length
7	4.1 3.4 2.7 3.1	41	5.31	48.78	2.41	4
8	2.6 1.4 4.2 3.2	42	5.44	47.61	2.35	4
9	4.1 1.3 3.1 2.5	42	5.44	47.61	2.35	4
10	4.1 1.3 3.1 4.2	41	5.31	46.34	2.29	4
32	4.1 1.3 3.1	54	7.00	46.29	2.28	3
11	2.6 1.5 4.2 3.2	44	5.70	45.45	2.24	4
12	2.1 1.5 3.1 2.4	42	5.44	45.23	2.23	4
13	1.3 1.5 1.1 4.2	40	5.188	45.00	2.22	4
1	1.3 3.1 2.4 4.2 2.5	54	7.004	44.44	2.19	5
14	1.3 1.5 2.7 4.2	45	5.837	44.44	2.19	4

It can be found from the above table that there is attribute union between rules. Therefore, if rules are directly mapped to products for weighted attribute value calculation, it will cause repeated calculation. Therefore, this paper defines the following calculation rules:

Step1 matching the maximum k-itemset rule that the product can correspond to

Step2 for products that correspond to multiple maximum k-itemset rules, select the rule that retains the maximum promotion

Step3 deduct the attributes involved in the corresponding k-itemset rule from the product attribute set

Step4 make $k=k-1$, repeat step1 to Step3 until no corresponding rule is found

Calculate the sum of the positive influence of the final retention rule $\alpha_{c_{TID=m}}$ (that is, the extra lift $\text{Lift}(c_{TID=m}) - 1$) as the corresponding attribute value of the product.

4. Calculation Method of Product Proximity Substitution Rate

This paper follows the idea of product proximity substitution in the location selection model and the exogenous demand model, that is, when the product is not selected or out of stock, it will only replace the two products in the nearest range. Suppose that the enterprise selects n products under a series of full category strategies, and the product set is recorded as $\{s\}$, where the weighted attribute value of the product has been obtained, and the attribute weighted value of product I is defined as $\alpha \setminus u I$.

In this paper, the products are sorted according to the weighted attribute value and renumbered, so that for product I , if $J > I$, then $\alpha_j > \alpha_i$, so for product I , there are two products in its adjacent range (when $i=1$ or N , there is only one adjacent product). And the academic circles' practice of dividing substitution into category driven stage and inventory driven stage is consistent. This paper also divides substitution into these two stages:

Phase I: category driven substitution

This paper continues the practice of K ok and Fisher et al., and uses the following method to calculate the replacement rate of category driven stage. Define β_{ij} as the replacement probability of product I when product J is not selected in the current period. Define the expected demand level of product I as d_i , which can be obtained from the sales volume of product I and other products in the near future (K ok, 2003). The customer's willingness to replace is δ . This parameter indicates the probability that the customer is willing to continue to consume in the store when he finds that product I is no longer sold. This probability can be estimated by the customer's loyalty to the store P_r . at this time, the estimated value of β_{ij} can be expressed by the following formula:

$$\beta_{ij} = \delta \frac{d_i}{\sum_{k \in n \setminus \{j\}} d_k} = P_r \cdot \frac{d_i}{\sum_{k \in n \setminus \{j\}} d_k} \quad j = \begin{cases} i + 1 & \text{for } i = 1 \\ i - 1, i + 1 & \text{for } 1 < i < n \\ i - 1 & \text{for } i = n \end{cases} \quad (6)$$

Phase II: inventory driven substitution

This stage is the replacement probability of product I for product J when product J is out of stock. The replacement rate at this stage is defined as α_{ij} . Referring to the practices of Gao Junjun and Gao and Shi, this paper defines the calculation formula of the replacement rate at this stage as follows:

$$\alpha_{ij} = \begin{cases} \delta, & \text{for } i = 1, j = i + 1 \\ \left(1 - \frac{|\alpha_i - \alpha_j|}{|\alpha_{i+1} - \alpha_{i-1}|} \right) \times \delta, & \text{for } 1 < i < n, j = i - 1, i + 1 \\ \delta, & \text{for } i = n, j = i - 1 \end{cases} \quad (7)$$

The significance of $\delta(P_r)$ is consistent with the estimation method and category driven stage. By changing the value of substitution intention δ , we can simulate the analysis of No. Because δ can only reflect the probability that the product is replaced as a whole, but can not reflect the replacement share of different products for the product, formula 7 uses the proximity of product attribute values to allocate the replacement share. The specific meaning of this formula is: for products with two adjacent alternatives, the substitution rate between the two products with closer attribute values is greater. By using the distance weighted method to allocate the substitution share of the two adjacent alternatives, the substitution rate between the products can be estimated.

5. Summary

This paper presents a method for estimating the substitution rate based on product attribute analysis. This method can avoid the red and blue bus problem in the MNL model, and can calculate the weighted attribute value of products whose attributes do not meet the sequencing conditions. At the same time, it also takes into account the role of attribute combination, and can make up for the shortcomings of the existing substitution rate estimation methods. The innovations of this paper are as follows: (1) from the perspective of product attribute combination, in view of consumers' preference for attribute combination, data mining method is used to find out the effective attribute set that can significantly affect consumers' decision-making behavior, so as to calculate the weighted attribute value of the product, and then the replacement rate estimation method of this kind of product is given. This paper can provide some reference for the current online retail enterprises to make operational decisions. In the future, the establishment of category planning joint decision-making model can be further considered.

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